

Week 10
MATH 34A
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3. A street light is at the top of a 19 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 7 ft/sec along a straight path.
- (a) How fast is the tip of her shadow moving along the ground when she is 45 ft from the base of the pole?
 - (b) How fast is the length of her shadow increasing?

9. Centerville is the headquarters of Greedy Cablevision Inc. The cable company is about to expand service to two nearby towns, Springfield and Shelbyville. There needs to be cable connecting Centerville to both towns. Centerville is located at $(7,0)$ in the xy -plane, Springfield is at $(0,3)$, and Shelbyville is at $(0,-3)$. To save on the cost of cable, Greedy Cablevision wants to arrange the cable in a Y-shaped configuration, running cable from Centerville to some point $(x,0)$ on the x -axis where it then splits into two branches, one going to Springfield and one to Shelbyville.

Find the location $(x,0)$ that will minimize the amount of cable between the 3 towns and compute the amount of cable needed. Justify your conclusion by answering the following questions.

- (a) What function f must we minimize to solve this problem?
- (b) Where does f have a critical point?
- (c) Find f'' at this critical point. Note that it is a positive number, and hence, f has a minimum at this critical point.
- (d) From all of the above, what is the minimal length of the cable?

10. In a certain chemical reaction, substance A combines with substance B to form substance Y . At the start of the reaction, the quantity of A present is a grams, and the quantity of B present is b grams. Assume $a < b$ and $y \leq a$. At time t seconds after the start of the reaction, the quantity of Y present is y grams. For certain types of reactions, the rate of the reaction, in grams/sec, is given by

$$\text{Rate} = k(a - y)(b - y),$$

where k is a positive constant.

- (a) Sketch a graph of the rate against y . For what values of y is the rate nonnegative? Give your answer as a union of intervals, e.g., $(-\infty, -a] \cup (a, 2b)$.
- (b) Use your graph to find the value of y at which the rate of the reaction is fastest.

14. As a preparation for the long bright summer days, Dr. Acula plans to store gourmet plasma in closed tin cans that have the shape of a cylinder with volume V . As an environmentally conscious member of the community, he wants to use as little metal as possible.
- (a) What is the height $h(r)$ for the can with minimum surface area in terms of the radius r of the bottom?
 - (b) What is the minimum surface area for the can in terms of the portion size V ?